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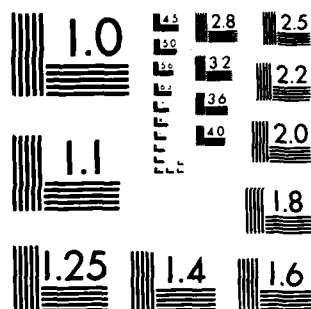
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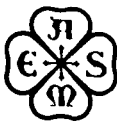
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ENERGETICS OF VORTEX RING FORMATION

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ABSTRACT

This paper presents an experimental investigation comparing the mass and energy content of fully formed laminar vortex rings in air with that of the original pulse which generated them for a variety of initial and boundary conditions. In particular, the fractional entrainment of mass and the partition of initial energy between kinetic energy of translation and kinetic energy of rotation is studied. It is found that a large degree of control can be exercised for the determination of the vortex energetics, as well as its final configuration. A technique is presented which enables calculation of kinetic energy of rotation from movie sequences. The ratio of characteristic translational speed to characteristic rotational speed is shown to be a useful parameter for correlation of data. Data on vortex size and speed are presented using this correlation and it is seen that all data, regardless of initial and boundary conditions, fall on a single curve. A theoretical curve is derived and it is seen that the data compare well with it.

NOTATION

- D = outside diameter of vortex
D_e = diameter of jet-pulse exit
D₀ = dimensionless vortex diameter, D/D₀
H = chimney height
E = kinetic energy
M = mass
P = linear momentum
t = time
T_p = duration of time of jet-pulse
u_j = jet pulse speed
 $\overline{u_j^2}$ = mean square jet pulse speed (Eq. 9)
U = translational speed of vortex
U* = dimensionless speed of vortex, (DU)/ $\overline{u_j^2}$ T_p
Γ = circulation
π = 3.1416
ρ = fluid density

Subscripts

- e = entrained fluid
i = initial pulse
j = jet
v = vortex

Superscripts

- R = rotational
T = translational
* = dimensionless quantity

INTRODUCTION

In direct-flow induction thrust augmentors, such as the ejector, the migration of finite sized eddies from the primary and secondary flows into each other gives rise to interface pressure forces which do work, thereby contributing a nondissipative component to the transfer of mechanical energy. In addition, there is a dissipative mode of energy transfer by turbulent shear stresses. As discussed by Foa (1), all direct flow induction thrust augmentors use to varying degrees each of these modes of energy transfer; to what extent depends upon the scale of the eddies. At one end of the spectrum is the ideal steady flow ejector where all energy is transferred by dissipative mixing, while at the other end is the ideal pulsating flow ejector where all energy is transferred by nondissipative pressure forces.

Lockwood and Patterson (2), Bernal and Borahia (3), and Viets, et. al. (4), as well as others have shown that appreciable improvements in ejector performance are possible when unsteady flow is present. In unsteady flow, the reversible work of interface pressure forces is utilized, particularly with pulsating flow ejectors. Foa (1) observed, however, that there may be appreciable losses inherent in pulsating flows. Firstly, there are the losses involved in generating the pulses. Secondly, there are losses resulting from the production of kinetic energy of rotation in vortex rings produced by the pulses. Foa (1) noted that the latter may be appreciable, and he further observed that this loss is present not only in the extreme case of the pulsating flow ejector, but whenever finite sized eddies, generated out of steady flows, participate in the induction process; the magnitude of the loss depending on the scale of the eddies.

In the present work, we attempt to quantify some of these observations by determining the fraction of the initial energy that is converted to kinetic energy of translation in the subsequent vortex. This is done for a variety of vortex generating conditions. To provide a better understanding of the effects of vortex generating conditions on the resulting vortex, size, speed, and entrainment are studied.

There is an abundant amount of literature on the structure, motion, and stability of vortex rings. An excellent review of the literature, as well as historical discussions, may be found in Widnall & Sullivan (5), Sullivan & Widnall (6), Maxworthy (7,8,9), Maxworthy & MacLachlan (10), Saffman (11), and others. A review will not be presented here. It is noted, however, that there is very little information available on the partition of mass and energy in vortices.

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THEORETICAL CONSIDERATIONS

Boundary Conditions and Initial Conditions

A sketch of the system under study is shown in Fig. 1. We consider that at time $t=0$, a pulse is initiated with velocity $u_j(t)$ for a duration T_p through a chimney of height H and diameter D_0 . Note that when $H=0$, we have a simple orifice. After a certain period of time, the pulse becomes a fully formed vortex ring of diameter D which propagates at a speed U . The nature of the vortex ring will depend on the boundary conditions (H, D_0) and on the initial conditions ($u_j(t), T_p$). Relevant mechanisms which cause this are discussed in Maxworthy (8) and Pullin (12).

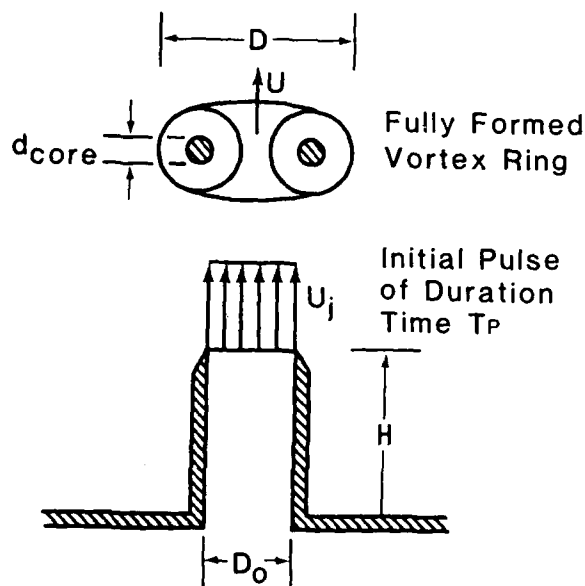


Fig. 1 - Definition sketch.

Since much of the vorticity essential to the existence of a vortex is generated in boundary layers prior to exit, different geometries will produce different amounts of vorticity. Furthermore, in the case of the simple orifice, vorticity of opposite sense is generated on the outside of the orifice by the entrained flow, thereby impeding the influx of entrained flow and diminishing the net available vorticity. These effects will result in the generation of vortices with different sizes and different propagation speeds, as well as differences in internal flow fields.

Conservation of Mass

Applying conservation of mass to the initial pulse emerging into a stagnant fluid and the resulting vortex ring:

$$M_i + M_e = M_v \quad (1)$$

or:

$$\frac{M_e}{M_v} = 1 - \frac{M_i}{M_v} \quad (2)$$

The mass of the initial jet is given by:

$$M_i = \frac{\rho \pi D_0^2}{4} \int_0^{T_p} u_j dt \quad (3)$$

The mass of the vortex can be measured by photographic observation, and u_j and T_p from hot-wire anemometer records.

Conservation of Linear Momentum

One may regard the mechanism of vortex formation as a simple inelastic collision between a primary slug of fluid and a stationary secondary fluid. The vortex ring is the product of the collision. If one neglects drag forces on the vortex, conservation of linear momentum may be applied:

$$P_i = P_v \quad (4)$$

or:

$$\frac{\rho \pi D_0^2}{4} \int_0^{T_p} u_j^2 dt = M_v U \quad (5)$$

Hence for conservation of linear momentum:

$$\frac{M_v U}{\frac{\rho \pi D_0^2}{4} \int_0^{T_p} u_j^2 dt} = 1.0 \quad (6)$$

M_v and U can be determined photographically.

Note that the mass of the vortex is the density times the volume. If we assume the vortex to be shaped as a disk with a flat top and circular ends such that the center of the circle is a distance $D/4$ from the vortex axis and the diameter of the circle is $D/2$, its volume is:

$$\text{VOLUME} = \frac{\pi}{32} \left(\frac{\pi}{2} + \frac{2}{3} + 1 \right) D^3 \quad (7)$$

Substituting (7) into (6) and rearranging, we obtain:

$$\frac{DU}{\int_0^{T_p} u_j^2 dt} = 2.4711 (D/D_0)^{-2} \quad (8)$$

Now:

$$\int_0^{T_p} u_j^2 dt = \overline{u_j^2} T_p \quad (9)$$

Hence:

$$\frac{DU}{\overline{u_j^2} T_p} = 2.4711 (D/D_0)^{-2} \quad (10)$$

or:

$$U^* = 2.4711 D^{*-2} \quad (11)$$

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A very useful physical interpretation of the parameter U^* may be obtained from the approximation that the initial pulse is a square wave of amplitude u_j (rms). In such case, the circulation, Γ , is given by:

$$\Gamma = \int \vec{u} \cdot d\vec{l} \quad T_p \quad (12)$$

hence:

$$U^* = \frac{U}{\Gamma/D} = \frac{\text{Translational Speed}}{\text{Rotational Speed}} \quad (13)$$

Thus, U^* may be regarded as a ratio of the vortex translational speed to a characteristic rotational speed.

Conservation of Energy

Conservation of energy may be written:

$$KE_i = KE_V^T + KE_V^R + \text{Losses} \quad (14)$$

where:

$$KE_i = \frac{\rho \pi D^2}{8} \int_0^{T_p} u_j^3 dt \quad (15)$$

and:

$$KE_V^T = \frac{M}{2} U^2 \quad (16)$$

The determination of rotational kinetic energy requires detailed information on the velocity field inside the vortex relative to a reference frame moving with the vortex. The velocity field is approximated by a Rankine combined vortex, viz., a rotational "core" where the fluid is in rigid body rotation, surrounded by an irrotational "free vortex". Thus, if one can determine the diameter of the core and the tangential velocity at the edge of the core, one could approximate the velocity field relative to the vortex and, hence, calculate the kinetic energy of rotation. The rotational core is assumed to be approximated by the envelope of the initially entrapped fluid. When a tracer is placed in the primary, one can observe this envelope clearly as the pulse emerges, flattens as a rivet, and surrounds a "bubble" of ambient fluid. Hence, the core can be made visible. The tangential velocity at the edge of the core can be estimated photographically as well. This will be discussed further.

EXPERIMENTAL METHODS

Vortex rings were generated in still air for various sizes of simple orifices and chimneys by means of a large loudspeaker energized by a step in voltage producing a controllable jet-pulse intensity.

The pulse intensity and duration were monitored by means of a hot wire anemometer. Oscillographic records of the anemometer trace enabled calculation of pulse mass, momentum, and energy by integration as previously discussed.

Flow visualization was achieved by introducing cigarette smoke in the primary through the loudspeaker enclosure, or by introducing paraffin smoke in the secondary fluid just outside the jet exit. Both methods of flow visualization were utilized to ensure that the visual boundaries of the vortex corresponded to the actual boundaries. Our results indicated that they did.

The motion and development of the vortex ring was established by means of both multiple-flash open-shutter photography with a General Radio Stratoskop and a 35 mm camera, and by means of high-speed movies with a HYCAM 16 mm cine camera. Chopped laser sheet illumination was also used. Irdema (13) discusses in detail the various experimental setups used.

RESULTS AND DISCUSSION

Qualitative Observations

Figure 2 is a two-flash open shutter photograph of a single vortex ring generated from a simple orifice. Figure 3 is a similar photograph of a vortex produced from a chimney of the same diameter. The maximum exit velocities were adjusted to be the same in both runs; however, the pulse duration for the chimney generated vortex is about 38% longer. To facilitate comparison, the time between exposures is the same and the length scales are approximately the same in both figures.

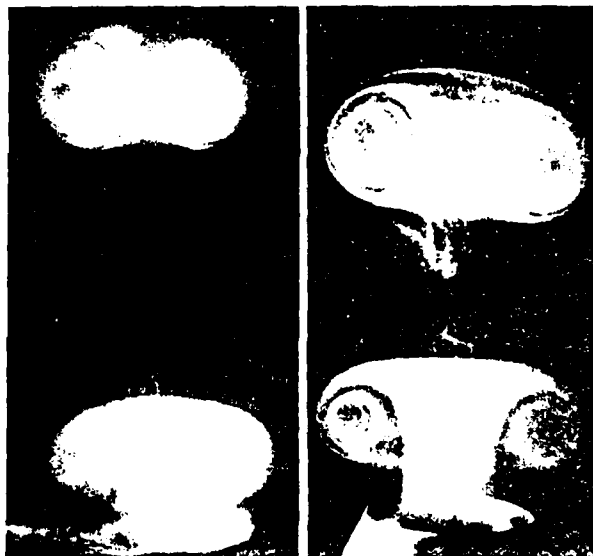


Fig. 2 (left) - Multiple flash photograph of an orifice generated vortex.

Fig. 3 (right) - Multiple flash photograph of a chimney generated vortex.

Comparison of the two figures reveals several features:

1. Size: The vortices produced with the chimney are appreciably larger than those produced with the simple orifice. The difference is on the order of 25%.
2. Translational Speed of Ring: Note the larger distance travelled between exposures in the orifice produced of Fig. 3. This indicates that the vortices produced by the orifice propagate considerably faster than those produced by the chimney for a given pulse jet strength and jet diameter. The difference is on the order of 30-35%.
3. Entrainment: Note the very clear spiral entrainment patterns of Fig. 3 as well as the large number of rings as compared with those of Fig. 2. This shows that the slower moving,

larger vortices tend to entrain more ambient fluid.

4. Formation Characteristics: Comparison of Figs. 2 and 3 shows that the orifice generated vortices tend to form very rapidly whereas the chimney generated vortices travel farther downstream before they are fully developed.

In Figs. 4 and 5, data taken from high speed movies of several vortices are summarized, again highlighting the comparison between orifice-generated and chimney-generated vortices.

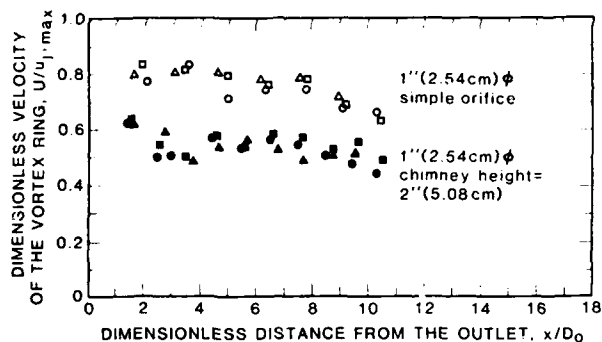


Fig. 4 - Comparison of vortex translational speed vs. distance from outlet for a simple orifice (unfilled symbols) and a chimney (filled symbol), both with similar initial conditions (symbol shapes refer to different vortex ring).

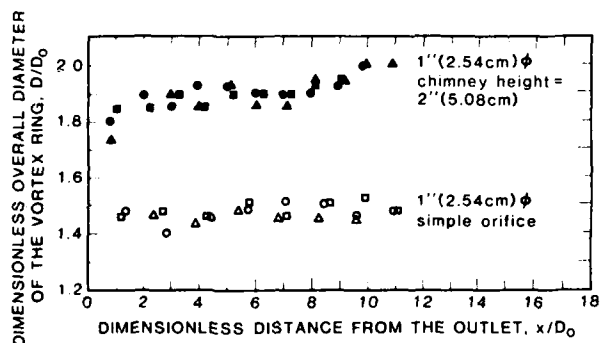


Fig. 5 - Comparison of vortex diameters vs. distances from outlet for a simple orifice (unfilled symbols) and a chimney (filled symbols), both with similar initial conditions (symbol shapes refer to different vortex ring).

Figure 4 further shows that, upon emerging, the vortex experiences a brief period of deceleration and then propagates at a relatively constant speed. Figure 5 shows that after forming, the vortices change little in size. Thus, in air, the deceleration and growth of the vortex is less pronounced than in water, as reported by Maxworthy (7).

Further comparisons were made between vortices formed from chimneys of varying height, while maintaining equal maximum jet exit speeds. The resulting effects on the vortex configuration was observed to be small.

Consider a pulse of a given amount of kinetic energy injected into still ambient fluid. If appreciable entrainment occurs, this kinetic energy must be shared by a greater amount of fluid mass, hence, one would expect a slower vortex translation. Furthermore, the vortex possesses both translational and rotational kinetic energies. Hence, if the vortex kinetic energy of rotation is very high, one might expect a slower translating vortex since more of the allotment of kinetic energy is absorbed in the rotational mode.

The clear, delicate spiral structure of the chimney-generated vortices of Fig. 3 indicates considerable entrainment. Since the spiral structure persists many diameters downstream, it appears that mixing within the vortex is very slow. The structure of the orifice generated vortex of Fig. 2, however, demonstrates much less entrainment as suggested by its more homogeneous appearance.

Although there are several differences in the fluid mechanics of the orifice and chimney vortices, a most significant one is that the ambient air enters the latter more freely, avoiding the boundary layer, during the initial stages of formation. This is supported by the weak dependence of vortex behavior on chimney height.

Correlation of Fully Developed Vortex Data

The data of this study is shown plotted with the theoretical prediction, (11), in Fig. 6. It is seen that the data indeed runs near to the ideal curve.

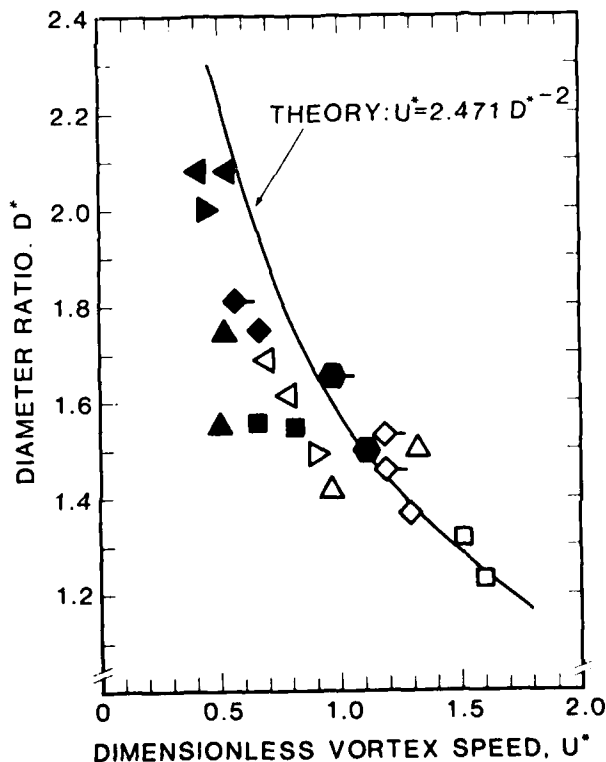


Fig. 6 - Ratio of vortex diameter to exit diameter vs. dimensionless vortex speed, U^* . Filled symbols are for chimneys & unfilled symbols are for orifices. (See Table 1 for meaning of symbols.)

Since momentum is not, in fact entirely conserved due to drag forces on the vortex, and the vortex is not precisely a disc of the form described in the model, one should consider the theoretical curve as an approximation. That most of the data appear below the curve suggests the effects of drag on the vortices.

TABLE 1 - Experimental Conditions of Figures 6, 7, and 8

Symbol *	Exit Diameter, D_0 in (cm)	Maximum Pulse Speed in/s(cm/s)
Δ	0.75 (1.91)	33 (83.82)
∇	0.75 (1.91)	45 (114.30)
\square	0.75 (1.91)	62 (157.48)
\diamond	1.00 (2.54)	33 (83.82)
\circ	1.00 (2.54)	45 (114.30)
\times	1.00 (2.54)	62 (157.48)
\bullet	1.50 (3.81)	45 (114.30)
\circ	1.50 (3.81)	62 (157.48)

*Filled symbols are from 2" (5.09 cm) high chimneys. Unfilled symbols are from simple orifices.

Figure 6, in conjunction with (10), does show that for a given diameter ratio, D^* , the vortex speed will vary as the mean square pulse intensity, and that for a given pulse intensity, the vortex speed varies inversely as the cube of the vortex diameter. The graph also shows the previously discussed result that the chimney vortices are larger and slower than the orifice vortices.

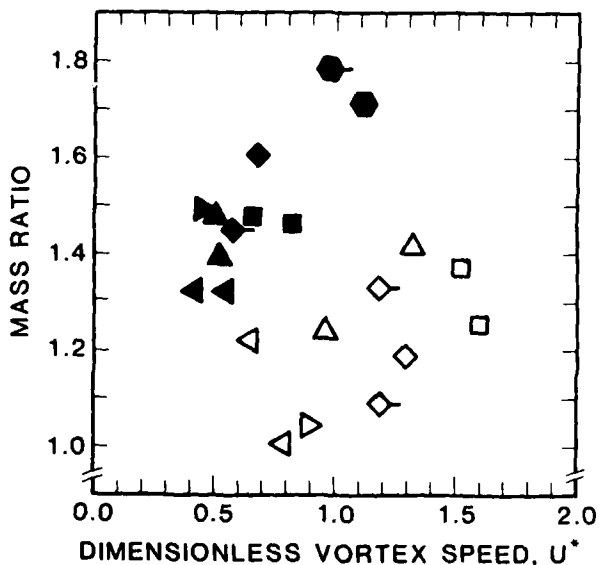


Fig. 7 - Ratio of vortex mass to initial pulse mass vs. Dimensionless vortex speed, U^* . Filled symbols are for chimneys and unfilled symbols are for orifices. (See Table 1 for meaning of symbols)

Entrainment

In Fig. 7 is shown the results of the experiments on entrainment and calculations as described in (2) and (3). It is readily noted that the chimney vortex data lie above and to the left of the orifice vortex data. This is consistent with the previous qualitative observations. For a given speed, diameter, and circulation, the chimney vortex will entrain much more fluid and it

will have originated from a smaller exit. Likewise, for a given percentage entrainment, the orifice vortices will be faster with less circulation than the chimney vortices.

Energetics

In Fig. 8 is shown the ratio of translational kinetic energy of the vortex to the initial kinetic energy of the jet vs. U^* . Although there is substantial scatter, all data appear to lie on a single curve. The chimney vortices show a much lower fraction of translational kinetic energy than the orifice vortices. As indicated by (14), this suggests that the chimney vortices have proportionately more kinetic energy of rotation. Note that this observation is consistent with the physical interpretation of U^* as being the ratio of translational to rotational characteristic velocities, since low U^* should imply high rotation and vice versa.

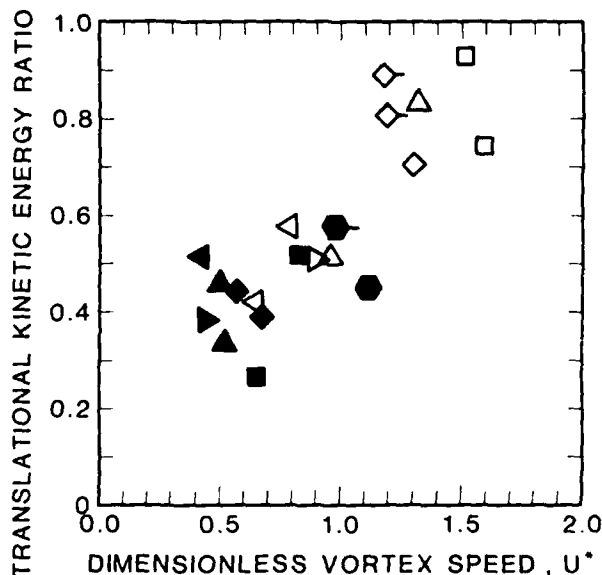


Fig. 8 - Ratio of translational kinetic energy of vortex to the initial kinetic energy of the pulse vs. dimensionless vortex speed, U^* . Filled symbols are for chimneys and unfilled symbols are for orifices. (See Table 1 for meaning of symbols)

It is interesting to note that, as predicted by Foa (1), for certain types of vortices, substantial fractions of the initial kinetic energy of translation are indeed lost. For some of the chimney vortices, the loss is of the order of 60-70%. However, for some of the orifice generated vortices, the loss is considerably smaller, as little as 10%.

Another interesting observation is that if the pulse duration is very short, vortices with high translational kinetic energy predominance may be formed, since short pulses lead to larger values of U^* . This is a significant consideration for ejector technology.

As indicated by (14), however, it remains to be determined how much of the lost energy from the initial pulse is in kinetic energy of rotation, and how much is in dissipation. To get some insight into this, an attempt was made to estimate the kinetic energy of rotation by the method described in Theoretical Considerations, viz., by tracing the smoke filament rolling up



Fig. 9 - Movie sequence of smoke filament in the process of surrounding the vortex core.

around the vortex core. A typical movie sequence is shown in Fig. 9 where the rolling up process can clearly be seen. As a fraction of the initial pulse kinetic energy, it was calculated that for a vortex produced by a chimney of 2" (5.08 cm) height and 1" (2.54 cm) in diameter with an initial rms pulse intensity of 25.1 in/s (63.8 cm/s), and a pulse duration of 40 ms, that the distribution of energy in the vortex is 53% translational, 18% rotational, and 29% dissipation. Again, it should be emphasized that the measurement of kinetic energy of rotation by this method is very crude and involves many assumptions, as previously outlined. Yet, that the result is in the right order of magnitude suggests that the method merits further development. However, the 18% figure for kinetic energy of rotation is probably quite low.

CONCLUDING REMARKS

This work has shown that:

1. The dimensionless translational to rotational speed ratio, U^* , is a very useful parameter for correlating vortex ring data.
2. Exit boundary conditions can have a very large effect on the entrainment and the partition of kinetic energies of translation and rotation.
3. Vortices with minimal amounts of kinetic energy of rotation can be generated by proper attention to boundary conditions and by using pulses of extremely short duration.

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